Estimating beta for Exxon Mobil

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1 Calculating log-returns

We can learn from graphs (Figure 1)[1] two things at least. **First of all**, time serious of returns have non constant sigma that varies heavy from time to time with volatility clustering at some points in time.

Secondly, if we look more deeply into distribution of returns we will find out that both market and Exxon Mobil log returns are not normal distribution is symmetric (almost zero skewness), but it has a sharper "peak" and fatter "tails", because kurtosis is 5,7 and 6,3 for Market Index and Exxon Mobil correspondingly. Almost zero p-value of Jargue-Bera test indicates that distribution of returns is indeed not normaliny.

2 Scater plot y against x

As we can see on Figure 2, y - excess return of exxen mobil over risk free rate is directly linked to the excess market return over risk free rate. This implies that risk premium on exxon mobil shares is directly linked to market risk premium.

Model we estimate model:

$$y_t = \alpha + \beta \cdot x_t + \varepsilon_t$$

where y is a difference between exxon mobil log-return and risk free rate, x - difference between market log-reutrn and risk free rate.

3 Regression y against x

In the table we see that estimate of a constant isvery close to zero. More over very low t-statistics and p-value of almost 95% indicates that we cannot reject hypotheses that constant is equal zero at almost 95% level. This, in turn implies that $\alpha = 0$ - exxon mobil shares has null expected excess return.

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[1] All the figures are in the end of the text.

Variable	Coefficient	St.error	p-value
Constant	0.002960	0.039866	0.9408
X	0.655388	0.032135	0.000000
R2	0.24		
Adj R2	0.24		
S.E.of Reg.	1.44		
SSR	2702		
F-stat	415.94		
P-value(F-st)	0.00		
number obs.	1305		

4 Interpretation of regression results for x

As we can learn from the table - x is significantly important in the regression. t-statistics of 20.39 and p-value of almost zero indicates that we cannot accept hypotheses that x's coefficient estimate $\widehat{\beta}$ is equal to zero or that the estimate ($\widehat{\beta}$) can be that large given if $\beta = 0$. That means that x - market risk premium significantly explains the y - exxon mobil risk premium.

5 Test if $\beta < 1$

The interpretation of $H_0 : \beta < 1$ means that exxon mobil systematic risk is smaller than market systematic risk. More formaly we can test with hypotheses using one-tail test:

$$H_0 : \beta = 1$$

$$H_1 : \beta < 1$$

The null hypothese is rejected if $t < -t_{\alpha,n-2}$. Estimated t-statistics is equal to:

$$\frac{\widehat{\beta} - \beta}{\sqrt{\widehat{V}(\widehat{\beta})}} = -10.72$$

$$t_{0.05,1305-2} = 1.64$$

As $t < -t_{\alpha,n-2}$,we cannot accept null hypothes that $\beta = 1$ against $\beta < 1$ at 5% confidience level.

6 Using subsample

The main difference in estimates in the two models (unrestricted and restricted) comes in the estimate of β. In the unrestricted model it's estimate is 0.65, while for the restricted - 0.75. This diffrence might be random, i.e. it can be explained by the certain realisation of the random error in equation ε_t. Or it might be explained by a structural break at some point - or in other words, company's share systematic risk can increase in time. This in turn might be captured by the higher β estimate of 0.75.

Variable	Coefficient	St.error	p-value
Constant	0.0022269	0.045453	0.6246
X	0.746602	0.057576	0.000000
R2	0.39		
Adj R2	0.39		
S.E.of Reg.	0.73		
SSR	140		
F-stat	168		
P-value(F-st)	0.00		
number obs.	263		

7 95% Confidence interval

$$-t_{\frac{\alpha}{2},n-2} \cdot \sqrt{\widehat{V}(\widehat{\beta})} + \widehat{\beta} < \beta < \widehat{\beta} + t_{\frac{\alpha}{2},n-2} \cdot \sqrt{\widehat{V}(\widehat{\beta})}$$

$\alpha = 0.05, n = 263, t_{0.025,263-2} = 1.97$. So the 95% confidence interval is:

$$0.63 < \beta < 0.86$$

We can see that initial estimate of 0.65 lie within this confidence interval, but almost at the left margin.

8 Testing $H_0 : \alpha = 0$

The p-value of 0.6246 suggest that we cannot reject the null hypotheses $H_0 : \alpha = 0$ against $H_1 : \alpha \neq 0$ at the level of confidence of about 62% at most.[2] In this respect this result is the same is in the unrestricted model (3 question) - exxon mobil shares returns does not have excess returns after adjusting (correcting) for systematic risk.

9 Discussion

- **Constant** β. This approach is very useful when we either want to test for structural break in β over time or we have other information that company's shares systematic risk is changing relatively to the market risk in time. From this point of view, we may obtain unbiased estimate of the most current β, if a structural break took place.

- **Accuracy.** Using a subsample may have different effects on accuracy of estimates. First of all, if we have a structural break, then using subsample will lead to unbiased estimate of β and it's correct standart error. This error can be even lower than the s.e. for the full sample. The exent of difference depends on the properties of structural break. In our case, we have almost identical s.e. for both unrestricted

[2]This automatically means that we cannot reject H_0 at 5,10,15,..,62% levels of confidence.

and restricted models, subsample contains only 235 observation out of 1305. From other point of view, we use less number of observation. That means that our estimate of β is more sensitive to particular realisation of error terms e. Less observations does not give an opportunity to exploit the consistency property of the O.L.S. estimator. Plus O.L.S. estimates are very sensitive to outliers. With smaller number of observation marginal effect of an outlier is higher for a smaller sample. All these concidiration also applies to the α estimate.

- **Properties of t-tests.** Subsample leads to a lower size of statistics (degree of freedom), while the $t_{\alpha,n-k}$ is derceasing in $n-k$. This means that given smaller sample critical value of t-statiscits is becoming higher - indicating that we need "more proof (evidence) with smaller sample".

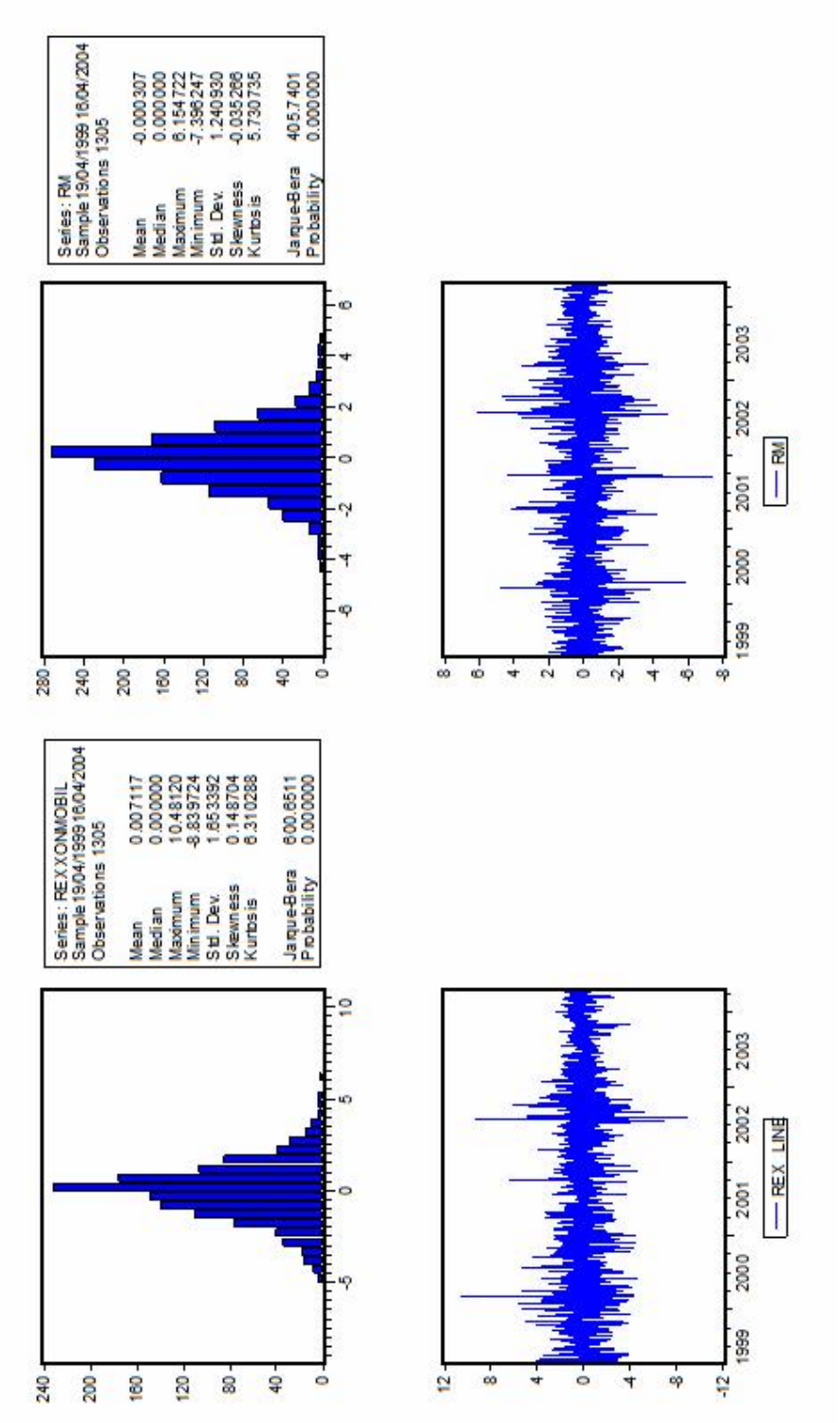

Figure 1: Market Rerurn and Return on Exxon Mobil shares: line and histogram

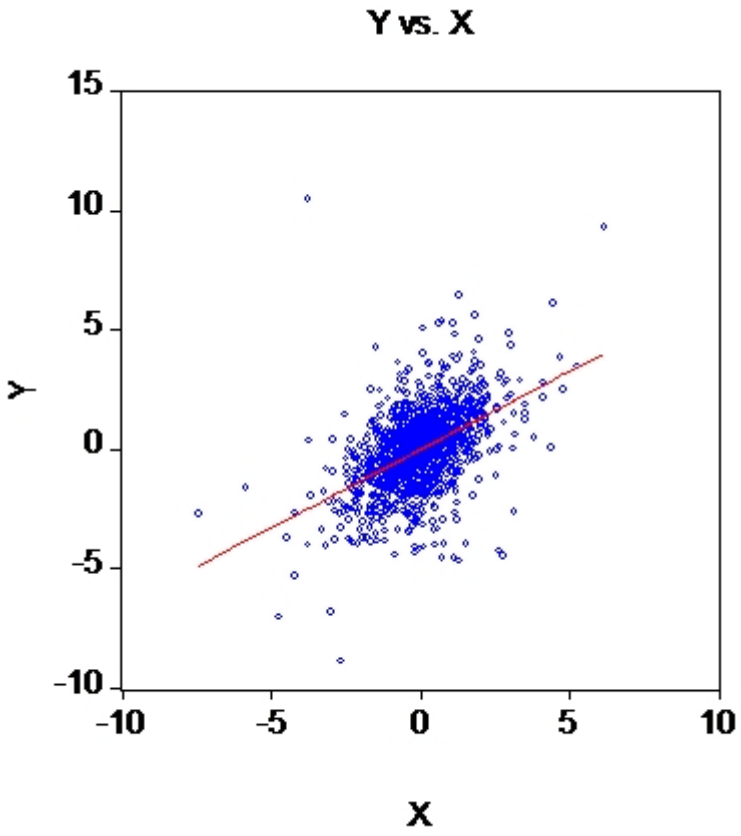

Figure 2: Market Rerurn and Return on Exxon Mobil shares: line and histogram